# Permanent-Income Theory with Risky Human Capital

Mark Huggett, Gustavo Ventura and Amir Yaron*

Work in Progress
Preliminary and Incomplete

Keywords: Permanent Income, Human Capital, Idiosyncratic Shocks.

*Affiliation: Georgetown University, Pennsylvania State University, and The Wharton School of Business and NBER respectively.
Corresponding author: Amir Yaron. Address: The Wharton School; University of Pennsylvania, Philadelphia PA 19104- 6367. E-mail: yaron@wharton.upenn.edu.

# 1 Introduction

The permanent-income hypothesis is the dominant framework for analyzing consumption and savings behavior. In this framework, consumption and savings behavior is the consequence of maximizing expected utility, given that earnings follow an exogenous process. The fact that earnings are exogenous is a great limitation of the theory. It means that the theory can provide little insight into important issues. For example, what accounts for important dimensions of inequality? A common view is that consumption and welfare are closely tied to lifetime earnings. Given this view, the natural questions are where do lifetime earnings differences come from and how can policy improve welfare through earnings. Clearly, the theory is not very helpful for answering these questions.

The next important step for permanent-income theory is to endogenize earnings. In this paper, we integrate a model of risky human capital into life-cycle, permanent income theory, and explore its implications on different fronts. There are a number of reasons for why such an integration is interesting.

> First, why build upon human capital theory? One alternative is a theory where wages are exogenous but labor is endogenous. Such a theory is not nearly as ambitious as a human capital approach because it leaves unexplained both the large differences in wages within a birth cohort and their evolution over the life cycle. Human capital theory explains these differences as reflecting differences in acquired skills.
>
> Second, one merit of human capital theory is that it is qualitatively consistent with a wide array of facts related to earnings and to skill acquistion. It explains why earnings profiles over the life-cycle are hump-shaped, why earnings profiles are more steeply sloped for people who choose more schooling, why some people choose more schooling than others and why skill accumulation is concentrated at young ages.
>
> Third, a model of risky human capital may lead to different answers to important questions. For example, what part of the variance of lifetime earnings is due to differences in initial conditions versus differences in shocks over the life cycle?[1] This is of interest as it relates to the relative importance of policies directed at differences in initial conditions (e.g. public education) versus differences in shocks over the life cycle (e.g. unemployment insurance). Models with

[1]This question has been analyzed by Keane and Wolpin (1997) within a career-choice model and by Storresletten et. al. (2004) within a permanent-income model.

exogenous earnings may overemphasize the importance of shocks because all the increase of earnings dispersion observed over the life cycle is accounted for with persistent earnings shocks (see Storresletten et. al. (2004)). Risky human capital models offer an additional source of earnings dispersion. Specifically, differences in learning ability, determined early in life, can lead to differences in the slopes of life-cycle-earnings profiles. Thus, a theory of risky human capital may lead to a greater importance of differences in initial conditions.

Fourth, a risky human capital model introduces some new questions into permanent-income theory. For example, are skills under accumulated under laissez faire?[2] If so, then welfare gains can come from policies that improve consumption smoothing and the allocation of human capital investment.

It is widely viewed that investments embodied in people are risky. We assume that the source of this risk is that an agent's skills are directly subject to idiosyncratic shocks. This translates into earnings risk as earnings equal the product of human capital, work time and a rental rate of human capital. This modeling choice follows Sargent and Ljungqvist (1998) and Krebs (2004).[3]

In the model an agent makes a consumption-savings choice each period as well as a decision on how to split available time between work and accumulating human capital. Absent human capital risk, there is a separation of time allocation decisions from consumption-savings decisions. An agent first maximizes the present value of earnings and then decides how to consume this present value over the lifetime.[4] With human capital risk this no longer holds as there are imperfect asset markets.

A natural first step for a risky human capital model is to see if it can replicate facts about how the earnings distribution evolves for a typical cohort as the cohort ages. Since the model has only idiosyncratic shocks, the evolution of the earnings distribution in the model is deterministic even though what happens to any particular agent is random. Thus, in US data we examine the effects of age on the cohort-specific earnings distribution, controling for time and cohort effects. We focus on the evolution of two central features of the earnings distribution: mean earnings and a measure of

[2]Benabou (2002) makes a start at quantitatively addressing this issue. Lehvari and Weiss (1974) establish theoretical results on how risk effects human capital investment in two-period models. When their two-period model is specialized to the framework considered in this paper, human capital investment is smaller with risk than without when agents are risk averse.

[3]We argue later in the paper that assuming that skills are directly subject to idiosyncratic shocks is, under some conditions, observationally equivalent to a theory where rental rates of human capital are subject to idiosyncratic shocks but where skills are not subject to shocks.

[4]This result relies on perfect capital markets and no leisure decision.

earnings dispersion. We find that mean earnings and earnings dispersion increase over the bulk of the working life cycle.

How does a risky human capital model explain these patterns? First, the increase in earnings is explained by the concentrated accumulation of human capital early in the working life cycle. The flattening out and fall in mean earnings later in life is explained by human capital depreciation together with the fact that optimal human capital investments fall at the end of the life cycle. Second, the increase in earnings dispersion is due to two forces. One force comes from the assumption that agents have different learning abilities. Different learning abilities imply that mean earnings profiles will have different slopes. Baker (1997) and Guvenen (2004) provide evidence for economically important differences in individual earnings growth rates over the life cycle. They find that these growth rates are negatively correlated with estimated intercepts. Human capital theory offers a natural interpretation. Earnings are low early in life, for those high-ability agents who optimally invest heavily early in life in the accumulation of skills. Earnings for these agents increase strongly later in life due to high skill accumulation and the shift of time from learning to earning. A second force comes from the assumption that agents are subject to idiosyncratic shocks to their human capital. This will lead even agents who are identical in terms of learning ability and initial human capital to experience differences in earnings over the life cycle.

We find that the risky human capital model is able to produce the general pattterns of mean earnings and earnings dispersion found in US data. This holds for human capital shock processes that range from having no risk to having considerable risk. To help separate these models we examine the empirical autocorrelation function of earnings growth rates. In US data the correlation of earnings growth rates one year apart is about $-0.3$ and beyond one year apart is approximately zero (see Abowd and Card (1989) among others). This evidence cannot be explained by the model in the absence of shocks as earnings growth is highly correlated at all lags. This is because the rise in earnings dispersion is explained entirely by differences in the shapes of age-earnings profiles across agents differing in learning ability. The simple models we examine, with independent shocks across periods, can produce the qualitative patterns observed in US data.

The framework we consider with iid shocks is also able to generate rising levels of consumption dispersion over the life-cycle, as it is the case in US data. In fact, one of the stochastic specifications generates a rise in consumption dispersion of about the same magnitude that Deaton and Paxson (1994) found. Finally, we investigate the contribution of initial conditions vs. life-cycle shocks in accounting for dispersion in lifetime earnings. Our preliminary findings indicate that initial conditions account for the bulk of the variance in the present value of earnings – about two thirds of it.

### 1.1 Related Literature

**[TO BE COMPLETED]**

## 2 A Model with Risky Human Capital

An agent's preferences over the life cycle are given by a calculation of expected discounted utility $E[\sum_{j=1}^{J} \beta^{j-1} u(c_j)]$. Each period the agent chooses how to split available time between time spent working and time spent producing human capital. The agent also makes a decision how to split available resources between consumption and savings. Savings are in a risk-free asset. A dynamic programming formulation of this decision problem is given below. $V_j(k, h; a)$ denotes the beginning of period value function of an age $j$ agent. The state variables are asset holdings $k$, the stock of human capital $h$ and an agent's immutable learning ability $a$. The value function after the last period of life is set to zero (i.e. $V_{J+1}(k, h; a) \equiv 0$).

$$V_j(k, h; a) = \max_{(c, k', l)} \{u(c) + \beta E[V_{j+1}(k', h'; a)]\}$$

s.t.

$$\begin{aligned} c + k' &\leq e_j + k(1 + r) && (1) \\ e_j &= w_j h(1 - l) && (2) \\ h' &= (f(h, l, a) + h)s' && (3) \\ k' &\geq -K_{j+1}(h, l; a) && (4) \\ l &\in [0, 1] && (5) \end{aligned}$$

Equation (1) describes the agent's budget constraint. Available resources equal the value of asset holdings $k(1+r)$ and labor earnings $e_j$. Equation (2) states that labor earnings equal the product of human capital $h$, time devoted to market work $(1-l)$ and a deterministic rental rate $w_j$. Using these resources, the agent chooses consumption and next period asset holdings. Equation (3) states that human capital next period depends on human capital $h$ this period, human capital production $f(h, l, a)$ and on a human capital shock $s'$ realized at the beginning of next period. The shock $s'$ lies in a finite set $S \subset R_+$, is independently and identically distributed over time and occurs with probability $\pi(s')$. Human capital production depends on current human capital

$h$, time devoted to human capital production $l$ and an agent's immutable learning ability $a$. An agent faces a borrowing limit $K_{j+1}(h, l; a)$ which asset holding cannot fall below. The next subsection describes the restrictions on the borrowing limit.

## 2.1 Borrowing Constraint

The borrowing limit $K_{j+1}(h, l; a)$ is set equal to the maximum an agent can borrow while still being able to repay the loan with certainty by the end of life. This is the maximum of the realized present value of earnings in the worst possible realization of human capital risk over the life cycle. We note that this is a generalization of the "natural" borrowing constraint considered in permanent-income theory (e.g. Miller (1976) and Schechtman (1976)). This borrowing limit is endogenously determined as it responds to future earnings prospects through the agent's current age $j$, human capital $h$, learning ability $a$ and work decision $l$. The maximization in the problem below, which defines this borrowing limit, is over time allocation profiles $\{l_{j+1}, ..., l_J\}$ in the worst possible realization of human capital risk. It is understood that human capital in period $j+1$ which enters into this definition is given by $h_{j+1} = (f(h, l, a) + h)\underline{s}$, where $\underline{s}$ is the worst possible (i.e. the smallest) human capital shock.[5]

$$K_{j+1}(h, l; a) \equiv \max \sum_{i \geq j+1} w_i h_i (1 - l_i)/(1 + r)^{i-j}$$

$$s.t. \;\; h_{i+1} = (f(h_i, l_i, a) + h_i)\underline{s}$$

## 2.2 Observational Equivalence

The model presented above has idiosyncratic shocks to human capital and no shocks to the rental rate. How would a model with no human capital shocks but with idiosyncratic rental rate shocks differ? We argue below that these two models are observationally equivalent for at least some specifications of the human capital production function.

[5] The borrowing limit can be described recursively, where $h' = (f(h, l_j, a) + h)\underline{s}$ and $h'' = (f(h', l_{j+1}, a) + h')\underline{s}$:

$$K_{j+1}(h, l_j; a) = \max_{l_{j+1}} (1 + r)^{-1}[w_{j+1}h'(1 - l_{j+1}) + K_{j+2}(h'', l_{j+1}; a)]$$

This problem is essentially the Ben-Porath (1967) model. With the parametric assumptions made in Huggett et. al. (2003 Prop. 1), the solution is known.

For example, assume that the production function $f(h,l,a) = hg(l,a)$ is linear in $h$. The human capital shock model is specified by $\{w_j, s_j\}_{j=1}^J$, whereas the rental rate shock model is specified by $\{\hat{w}_j, \hat{s}_j\}_{j=1}^J$. Given a deterministic sequence $\{w_j\}_{j=1}^J$ and a stochastic process $\{s_j\}_{j=1}^J$, choose the rental rate shock model so that $\hat{s}_j \equiv 1, \forall j$ and so that $\hat{w}_1 \equiv w_1$ and $\hat{w}_j \equiv w_j s_2 ... s_j$ for $j \geq 2$. Define the budget constraints in the two models to be stochastic processes for $\{c_j, k_j, l_j, e_j\}_{j=1}^J$ such that there is a human capital process for which budget equations (1)-(5) hold, given the respective processes for rental rates and human capital shocks. The budget sets in the two models are then identical by construction. Intuitively, human capital shocks are simply relabeled as rental rate shocks. Since the objective function in both models is the same, then the best choice set in the two models coincide. This establishes that for a given human capital shock model one can find rental rate shocks for which both models produce the same observables.[6]

We take away two points. The first is that one cannot use data on earnings, consumption, time allocation and asset holdings to distinguish these two models. By extension, one cannot distinguish either model from a hybrid model with a combination of rental shocks and human capital shocks. The second point concerns interpretation. One might think that it is intuitively plausible that there is temporary variation in individual rental rates (say due to local labor market conditions) and permanent variation in skills (say due to skills becoming obsolete). Such a perspective may be useful for thinking about the origin of shocks or for thinking about sources of temporary versus persistent variation in earnings. Observational equivalence tells us that, within this model, one cannot move beyond the issue of perspective to the issue of identification, absent strong a priori assumptions on the nature of these shocks.

# 3 Parameter Values

Table 1 specifies parameter values for the risky human capital model. Agents live a lifetime of $J = 61$ model periods which corresponds to real-life ages 20 to 80. We assume that the working life cycle ends in some retirement period $R$ that occurs before the terminal age $J$. We set $R = 46$ so that retirement occurs at a real-life age of 65. In this retirement period the agent can no longer work and, thus, receives no labor earnings.

[6]To complete the observational equivalence argument, it remains to establish that given rental rate shocks $\{\hat{w}_j\}_{j=1}^J$ and $\hat{s}_j \equiv 1$ one can find human capital shocks such that the same holds. This can be done by choosing $w_j \equiv 1$ and $s_j \equiv \hat{w}_j / \hat{w}_{j-1}$ for $j \geq 2$.

Table 1: Parameter Values

| Definition | Symbol | Value |
|---|---|---|
| Model Periods | $J$ | $J = 61$ |
| Retirement Period | $R$ | $R = 46$ |
| Interest Rate | $r$ | $r = 0.04$ |
| Discount Factor | $\beta$ | $\beta = 1.0/(1+r)$ |
| Preferences | $u(c) = c^{1-\gamma}/(1-\gamma)$ | $\gamma = 2.0$ |
| Rental Rate Growth | $w_j = (1+g)^{j-1}$ | $g = .0014$ |
| Production Function Elasticity | $f(h,l,a) = a(hl)^{\alpha}$ | $\alpha \in [0.5 - 1.0)$ |

We set the discount factor $\beta$ and the real interest rate $r$ so that $\beta = 1/(1+r)$. Thus, absent human capital risk and binding borrowing constraints the consumption profile produced by the model would be flat. We set the real interest rate to $r = .04$. The period utility function is of constant relative risk aversion class, where the coefficient of relative risk aversion $\gamma = 2$.

We assume that the rental rate of human capital $w_j = (1+g)^{j-1}$ grows over the life cycle at a constant rate $g$. We set the growth rate $g$ equal to the average growth rate of mean cross-sectional earnings in the US over the period 1968-1992. We note that with stable demographics and a constant growth of the rental rate, the cross-sectional average earnings growth in the model economy is precisely equal to $g$. This assumes that human capital risk does not change over time. This last assumption is consistent with the goal of explaning the dynamics of the earnings distribution over the life cycle for a typical cohort as the cohort ages. Thus, the model is directed at explaining steady-state behavior.[7]

The human capital production function $f(h,l,a)$ is of the constant elasticity form explored by Ben-Porath (1967), where $\alpha$ is the elasticity parameter. Browning et.

[7]To associate growth in average earnings with deterministic growth in the rental rate we are making a strong assumption on the nature of idiosyncratic variation in rental rates and human capital shocks. Specifically, we are assuming that one is deterministic and one is stochastic and time invariant. We argued in section 2.2 that strong a priori assumptions are needed.

al. (1999) survey the literature that estimates this parameter. The estimates fall between 0.5 and 1.0. This literature abstracts from human capital risk and allows for only limited heterogeneity in initial conditions across agents.

# 4 Earnings Facts

In this section we ask whether the benchmark model can match facts on how the earnings distribution changes for a cohort as the cohort ages. The facts in question are mean earnings and dispersion in earnings, measured by the Gini coefficient. These facts are based on repeated US cross-section data on earnings.

We divide the parameters characterizing the model into two groups. The first group of model parameters were specified in Table 1. The remaining parameters characterize the human capital shocks and the initial joint distribution of human capital and learning ability.

We consider three different processes for human capital shocks. The set of shocks $S$ are the same for each process but the shock probabilities $\pi(s)$ differ. These three processes are listed below.

$$S = \{s_1, s_2, s_3\} = \{0.88, 0.98, 1.08\}$$

Case 1: $\pi(s_1) = \pi(s_3) = 0, \pi(s_2) = 1.0$

Case 2: $\pi(s_1) = \pi(s_3) = 0.20, \pi(s_2) = 0.60$

Case 3: $\pi(s_1) = \pi(s_3) = 0.40, \pi(s_2) = 0.20$

The shock processes differ in risk but have the same mean.[8] Case 1 is the deterministic version of the model. Under this specification, near the end of the life-cycle, mean earnings decline at a yearly rate of about 2%. Note that we progressively add risk under the stochastic specifications 2 and 3, by keeping the magnitude of the shocks constant but making the realizations of lowest and highest shocks more likely.

[8] At a later stage the geometric mean of the shocks could be selected to match the rate of decline of mean earnings in the data at the end of the working life cycle. The model implies that the gross growth rate of mean earnings approximately equals $(1+g)\bar{s}$, where $g$ is the growth rate of the rental rate and $\bar{s}$ is the geometric mean of the shocks. This follows as, under the model, human capital investment is approximately zero at the end of the working life cycle and, thus, earnings equal human capital times the rental rate. The mean of the shocks in the paper is not currently selected in this way.

For each shock process the initial distribution of human capital and learning ability is chosen to best match the US earnings facts, given all other model parameters. We choose the initial distribution under the assumption that it is bivariate log-normal. This distribution is characterized by 5 parameters.[9] To obtain the results we show below, we draw $20,000$ individuals at age 20 from any given choice of the distribution of initial human capital and learning ability. Using decision rules from the solution to the individual problem, we simulate for each of these draws life-cycle paths of human capital, consumption, labor earnings, etc. We subsequently use this large sample to compute all the model statistics we report.

**Mean and Dispersion in Earnings**

The results associated to the distributions that best reproduce the age-profiles of mean and dispersion are displayed in Figure 1 below. Figure 1 shows that each of these models is able to produce the qualitative properties of mean earnings and earnings dispersion found in US data. In particular, the model reproduces quite well the rise in earnings dispersion over the life-cycle in all cases.

[Insert Figure 1 a-b Here]

What are the properties of the initial distributions that best reproduce the earnings facts? Table 2 shows these properties for a curvature parameter $\alpha = 0.7$. We also report in the Table the a measure of goodness of fit (Mean Absolute Deviation).[10]

When shocks over the life-cycle are not present (Case 1), it is not problematic for the model to generate the earnings facts we focus on. We analyzed essentially this case in detail, the Ben-Porath model, in Huggett et. al. (2003). We demonstrated that this model can reproduce the earnings facts for the range values of the curvature parameter of the production function for human capital considered in Table 1. To deliver this result, learning ability differences are essential; without them, the model generates a profile of earnings dispersion that is decreasing as individuals age. But learning ability differences are not sufficient. As Table 2 indicates, learning ability must be positive correlated with initial human capital at the start of the life-cycle, implying that initial human capital must also differ. In the absence of these latter differences, the model generates, counterfactually, a sharp U-profile in earnings dispersion.

[9] In later versions we will also report results when we follow a non-parametric approach for the choice of the initial distribution.

[10] This is calculated as follows. Let $e_j$ and $\hat{e}_j$ denote mean earnings in the model and in the data respectively. Let $d_j$ and $\hat{d}_j$ denote earnings dispersion in the model and in the data respectively. Mean absolute deviation is calculated as $\sum_{j=1}^{R}[|\log(e_j/\hat{e}_j| + |\log(d_j/\hat{d}_j|]/(2R)$.

Table 2: Properties of Initial Distributions
($\alpha = 0.7$)

| Statistic | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| Mean Learning Ability ($a$) | 0.243 | 0.270 | 0.306 |
| Coefficient of Variation ($a$) | 0.363 | 0.297 | 0.247 |
| Mean Initial Human Capital ($h_1$) | 106.6 | 101.6 | 100.0 |
| Coefficient of Variation ($h_1$) | 0.451 | 0.427 | 0.386 |
| Correlation ($a, h_1$) | 0.754 | 0.781 | 0.772 |
| Mean Absolute Deviation (%) | 4.19 | 4.05 | 5.05 |

As we move from Case 1 to Case 3, the importance of idiosyncratic shocks increases as "good" and "bad" human capital shocks become more likely. As Table 2 shows, the initial distributions that best reproduce the earnings facts require higher levels of mean initial learning ability and lower levels of dispersion in it. Regarding initial human capital, we require lower mean levels and lower dispersion in relation to Case 1. What account for these changes in the initial distributions? Figure 2 a-b illustrate the consequences of adding shocks for a *given* initial distribution. The figure shows mean earnings and earnings dispersion for the initial distribution of Case 1, under the stochastic specifications for shocks of Case 1 and 2. The figures demonstrate that adding shocks to the model leads to a clockwise shift in the mean earnings profile, and to a lower value of earnings dispersion early in the life-cycle. This is accompanied by a steeper rise in dispersion as individuals age.

[Insert Figure 2 a-b Here]

When uninsurable shocks to human capital are present, risk averse agents shift away from human capital accumulation. Thus, everything else equal and relative to the absence of shocks, individuals' response to human capital risk dictates higher levels of earnings early in life and a lower growth rate in mean earnings. The net result is a clockwise movement in the mean earnings profile.[11] In terms of dispersion in labor earnings, note that human capital shocks are more important for agents of relative high learning ability. These agents are the ones who would allocate a

[11]This is effectively the central result of Lehvari and Weiss (1974) in a multiperiod setting. They showed in a two-period model that time into human capital production is smaller with human capital risk than without.

large fraction of time into human capital accumulation, absent shocks. Consequently, earnings dispersion is lower at the start of the life-cycle, and shows a steeper rise afterwards.

To reproduce the earnings facts, initial conditions need to be adjusted. Higher mean levels of learning ability and lower levels of initial human capital increase the returns to human capital accumulation. In consequence, they reduce the levels of labor earnings early in life while increasing growth in mean earnings. Likewise, lower levels of dispersion in learning ability reduce the fraction in the population with relatively high levels of learning ability, leading to a reduction in the slope in the earnings dispersion profile as the data requires.

**Growth Rates in Individual Earnings**

It is interesting to examine the autocorrelation function of earnings growth rates in these three models. It has been previously argued that the empirical autocorrelation function is at odds with that produced by economic or statistical models of earnings growth with an important role for persistent differences in individual earnings growth rates (e.g. Abowd and Card (1989, p. 427-8)). The argument is that models with differences in individual growth rates should have positive autocorrelations beyond a one year lag. Abowd and Card (1989) find that in US data the correlation of growth rates with a one-year lag is about $-0.3$ and the correlations beyond a one-year lag are close to zero. Baker (1997) finds similar results.[12]

Table 3 presents the auto-correlation function for earnings growth for the three model economies, at ages 40 and 45. The deterministic model (Case 1) produces positive and high autocorrelations of earnings growth rates at various lags. The reason is that the only force for producing increasing earnings dispersion with age is differences in the slopes of age-earnings profiles. These differences in age-earnings profiles produce strong persistence in earnings growth rates. When human capital is risky the increase in earnings dispersion is due to two forces: differences induced by learning ability and differences induced by shocks. Table 3 shows that when human capital is risky the correlation of earnings growth rates is in consistent with the data: it is negative for a one-year lag and close to zero afterwards. Moreover, the one-year lag correlation tends to fall as human capital risk increases. Thus, human capital risk

[12]Guvenen (2004) argues that (1) there is evidence for economically important differences in earnings growth rates across individuals in US data and (2) a statistical model of earnings with this feature produces an autocorrelation function of earnings growth rates similar to that in US data. We provide an economic model that tries to make sense of the earnings observations in Figure 1 and the autocorrelation properties of earnings growth rates.

acts to counteract the strong persistence arising from differences in learning ability. It offers a natural channel to reconcile theory and observations.

Table 3: Correlation of Growth Rates $\alpha = 0.7$

| Statistic | Age $(j) = 45$ | Age $(j) = 40$ |
|---|---|---|
| Case 1: $\pi(s_1) = \pi(s_3) = 0.0, \pi(s_2) = 1.0$ | | |
| Correlation$(z_j, z_{j-1})$ | 0.787 | 0.849 |
| Correlation$(z_j, z_{j-3})$ | 0.598 | 0.761 |
| Correlation$(z_j, z_{j-5})$ | 0.686 | 0.742 |
| Case 2: $\pi(s_1) = \pi(s_3) = 0.20, \pi(s_2) = 0.60$ | | |
| Correlation$(z_j, z_{j-1})$ | -0.164 | -0.132 |
| Correlation$(z_j, z_{j-3})$ | 0.018 | 0.017 |
| Correlation$(z_j, z_{j-5})$ | -0.0004 | 0.016 |
| Case 3: $\pi(s_1) = \pi(s_3) = 0.40, \pi(s_2) = 0.20$ | | |
| Correlation$(z_j, z_{j-1})$ | -0.144 | -0.181 |
| Correlation$(z_j, z_{j-3})$ | 0.0007 | 0.0008 |
| Correlation$(z_j, z_{j-5})$ | -0.003 | -0.010 |

$z_j \equiv (E_j/E_{j-1} - 1)$ and $E_j$ denote earnings growth rates levels at age $j$, respectively.

To provide some intuition on why the model with shocks is able to generate negative correlations in growth rates, consider the case of only "bad" shocks. We showed in Huggett et. al. (2003, Proposition 1) that earnings growth rates are always higher for agents that have lower human capital, holding age and learning ability fixed. Taking into account the fact that shocks are iid, consider the effects of a shock that destroys human capital at age $j$. This event (i) reduces earnings at $j$ creating a lower than otherwise growth rate from $j-1$ to $j$; (ii) induces higher human capital and earnings growth form $j$ to $j+1$. This results in earnings growth rates at $j+1$ and $j$ that are negatively correlated.

When there are "good" and "bad" human capital shocks, the above logic does not necessarily work. For our results to hold, it is important to bear in mind that the average realization of the the human capital shock is less than 1; we parameterize shocks so that the average decline in earnings near the end of the life-cycle is of about 2%. That is, "bad" shocks dominate in our case.

# 5 Implications of the Framework

We now report the implications of the model on two central issues. These relate to the dispersion in consumption over the life-cycle, and the relative importance of initial conditions in producing heterogeneity in lifetime earnings.

## 5.1 Consumption Dispersion

As documented by Deaton and Paxson (1994) and others, a key observation from US data is that consumption dispersion rises nearly monotonically over the life-cycle for a cohort as the cohort ages. Is the model with human capital risk consistent with these observations?

The model implications are displayed in Figure 3, when dispersion in consumption is measured by the variance of log-consumption. An important result emerges. In the absence of shocks, consumption dispersion is constant over the life-cycle, while it shifts counter-clockwise as the importance of shocks increases. For Case 2, the variance of log-consumption goes from 0.211 to 0.339 points; an increase of about 12.8 points. For case 3, respective increase is from 0.169 to 0.391 (about 22.2 points). Clearly, the model in the presence of risk can produce a substantial rise; for Case 3, the magnitude of the rise in dispersion is nearly the same as Deaton and Paxson (1994) found. Notice that this occurs *without* appealing to persistent shocks.[13]

[Insert Figure 3 Here]

In Case 1, the level of consumption at each age is constant and in proportion to the present value of individual earnings. This results in dispersion in consumption at all ages being the same and proportional to dispersion in the present value of earnings. Thus, the model in this case, while able to reproduce mean and earnings dispersion for a cohort, is not surprisingly at odds with the consumption data.

What accounts for the shift in the age-profile of consumption dispersion? Note first that as shocks become more important, individuals attempt to self-insure by

[13] Storesletten et. al. (2004) found that shocks to labor earnings with high persistence are necessary to produce a rise in consumption dispersion that resembles the actual one.

accumulating assets. Moving from Case 1 to 3 would lead to consumption tracking labor income more closely as self-insurance becomes harder. Under imperfect insurance then, keeping the distribution of learning ability and initial human capital constant, increasing levels of human capital risk would result in an increase in consumption dispersion at age 20 as well as an increase in the slope of the age-profile.

However, as human capital risk become more important, the level of dispersion in earnings driven by initial conditions falls. This leads to a drop in the the level of consumption inequality at the start of the life-cycle. Put differently, the amount of consumption variation across individuals that cannot be eliminated via asset trading is lower. In our calculations, this latter effect dominates at age 20. As we move from Case 1 to 3, the net result is then the counter-clockwise movement in the age-profile of consumption dispersion that Figure 3 illustrates; consumption inequality drops at age 20 and then display a steeper increase.

## 5.2 Importance of Initial Conditions

A fundamental question in the study of economic inequality pertains to the relative importance of initial conditions vs. shocks over the life-cycle in the determination of lifetime earnings. Our model provides a natural framework to investigate this issue; realized labor earnings are the result of human capital shocks and individual choices, which in turn depend on initial conditions as well as current and present shocks.

Preliminary results are shown in Table 4. We focus on the Present Value of labor earnings (PV) at age 20. The table shows dispersion in this variable (measured by the coefficient of variation), and a measure of the contribution of initial conditions to the total variance in PV. More specifically, we report the following. Let $\sigma^2$ be the total variance in the present value of earnings across individuals at age 20. Let $\sigma_{a,h_1}$ be the variance in the expected present value of earnings with respect to initial conditions. That is,

$$\sigma_{a,h_1} \equiv \text{var}[E(PV|a, h_1)],$$

where the variance is calculated with respect to the distribution of initial conditions, $F(a, h_1)$. Thus, the second row in the Table reports

$$\frac{\sigma^2_{a,h_1}}{\sigma^2}$$

Note first that dispersion in the Present Value of Earnings is approximately constant across different shocks specifications. This is not surprising since initial conditions are chosen to best reproduce average earnings and dispersion in earnings over the life-cycle. Second, naturally as we move from Case 1 to 3, the relative importance of initial conditions diminishes since shocks become more important. Quantitatively, at least about 2/3 of the total variance can be attributed to initial conditions (Case 3). That is, the bulk of dispersion in the present value of earnings in our examples is due to initial conditions.

Table 4: Dispersion in Present Value of Earnings

| Statistic | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| Coefficient of Variation | 0.533 | 0.550 | 0.544 |
| Contribution of Initial Conditions ($a$) | 1.0 | 0.83 | 0.66 |

These results are of interest in relation to other findings in the literature. Keanne and Wolpin (1997) in a model of career choice with accumulation of skills found that heterogeneity realized at age 16 accounted for about 90% of the variance of earnings as of age 26. Storesletten et. al. (2004) found that their estimate of fixed effects in individual earnings processes accounts for about half of the total variance in the present value of earnings, when idiosyncratic shocks are found to have high persistence. The estimate of about 2/3 we obtain for Case 3 is of special value since this case is able to generate a rise in consumption dispersion close to observations, under human capital shocks are identically distributed over time.

# 6 Other Issues

**[TO BE ADDED]**

# References


Abowd, J. and D. Card (1989), On the Covariance Structure of Earnings and Hours Changes, *Econometrica,* 57, 411-45.

Baker, M. (1997), Growth-rate heterogeneity and the Covariance Structure of Life Cycle Earnings, *Journal of Labor Economics,* 15, 338-75.

Benabou, R. (2002), Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?, *Econometrica,* 70, 481-517.

Ben-Porath, Y. (1967), The Production of Human Capital and the Life Cycle of Earnings, *Journal of Political Economy,* 75, 352-65.

Browning, M., Hansen, L. and J. Heckman (1999), Micro Data and General Equilibrium Models, in *Handbook of Macroeconomics,* ed. J.B. Taylor and M. Woodford, (Elsevier Science B.V, Amsterdam).

Cameron, S. and J. Heckman (1998), Life Cycle Schooling and Dynamic Selection Bias: Models and Evidence for Five Cohorts of American Males, *Journal of Political Economy,* 106, 262-333.

Deaton, A. and C. Paxson (1994), Intertemporal Choice and Inequality, *Journal of Political Economy,* 102, 437-67.

Guvenen, F. (2004), Learning your Earning: Are Labor Income Shocks Really That Persistent?, manuscript.

Heathcote, J., Storresletten, K. and G. Violante (2004), The Cross-Sectional Implications of Rising Wage Inequality in the United States, manuscript.

Heckman, J., Lochner, L. and C. Taber (1998), Explaining Rising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labor Earnings with Heterogeneous Agents, *Review of Economic Dynamics,* 1, 1-58.

Huggett, M., Ventura, G. and A. Yaron (2002), Human Capital and Earnings Distribution Dynamics, NBER working paper 9366.

Katz and Autor (1999), Changes in the Wage Structure and Earnings Inequality, in *Handbook of Labor Economics,*Volume 3A, 1463- 1555, eds. Ashenfelter and Card, (Elsevier Science, New York).

Keane, M. and K. Wolpin (1997), The Career Decisions of Young Men, *Journal of Political Economy*, 105(3), 473-522.

Keane, M. and K. Wolpin (2001), The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment, *International Economic Review*, November.

Kletzer, L. (1998), Job Displacement, *Journal of Economic Perspectives,* 12, 115-36.

Krebs, T (2004), Human Capital Risk and Economic Growth, *Quarterly Journal of Economics,* 118, 709- 744.

Levhari, D. and Y. Weiss (1974), The Effect of Risk on the Investment in Human Capital, *American Economic Review,* 64, 950-63.

Lochner, L. and A. Monge (2002), Human Capital Formation with Endogenous Credit Constraints, NBER working paper 8815.

Miller, B. (1976), The Effect on Optimal Consumpiton of Increased Uncertainty in Labor Income in the Multiperiod Case, *Journal of Economic Theory,* 13, 154- 67.

Mincer, J. (1997), The Production of Human Capital and the Life Cycle of Earnings: Variations on a Theme, *Journal of Labor-Economics,* 15 (1), Part 2.

Press, W. et. al. (1992), *Numerical recipes in FORTRAN,* Second Edition, (Cambridge University Press, Cambridge).

Pries, M. (2001), Uninsured Idiosyncratic Risk and Human Capital Accumulation, manuscript.

Ruhm, C. (1991), Are Workers Permanently Scarred by Job Displacements?, *American Economic Review,* 81, 319- 24.

Sargent, T. and L. Ljungqvist (1998), The European Unemployment Dilemma, *Journal of Political Economy,* 106, 514- 50.

Schechtman, J. (1976), An Income Fluctuation Problem, *Journal of Economic Theory,* 12, 218- 41.

Stevens, A. (1997), Persistent Effects of Job Displacement: The Importance of Multiple Job Losses, *Journal of Labor Economics,* 15, 165-88.

Storesletten, K., Telmer, C. and A. Yaron (2004), Consumption and Risk Sharing Over the Life Cycle, forthcoming *Journal of Monetary Economics.*

Figure 1-a

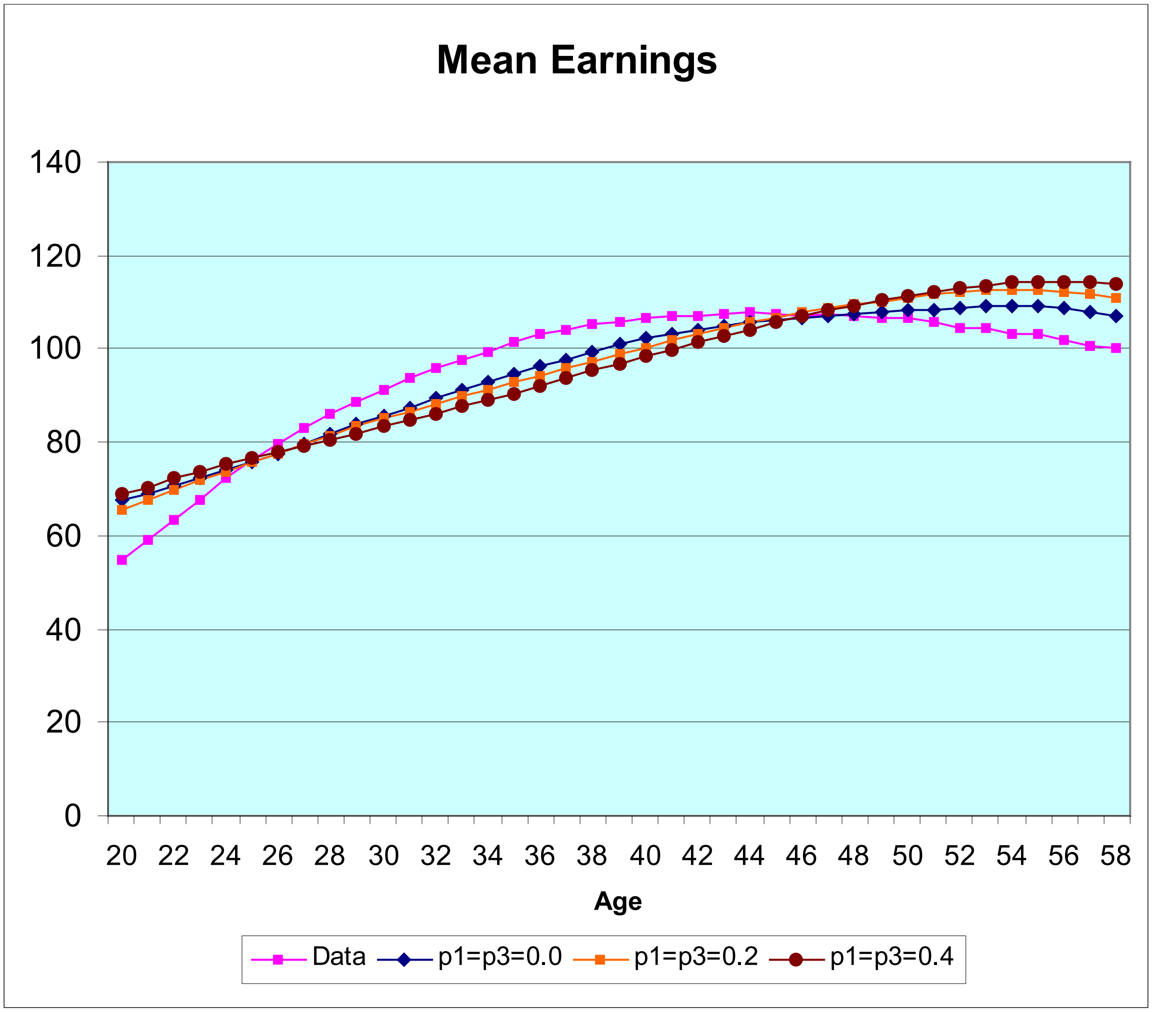

Mean Earnings
140
120
100
80
60
40
20
0
20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58
Age
Data
p1=p3=0.0
p1=p3=0.2
p1=p3=0.4

Figure 1-b

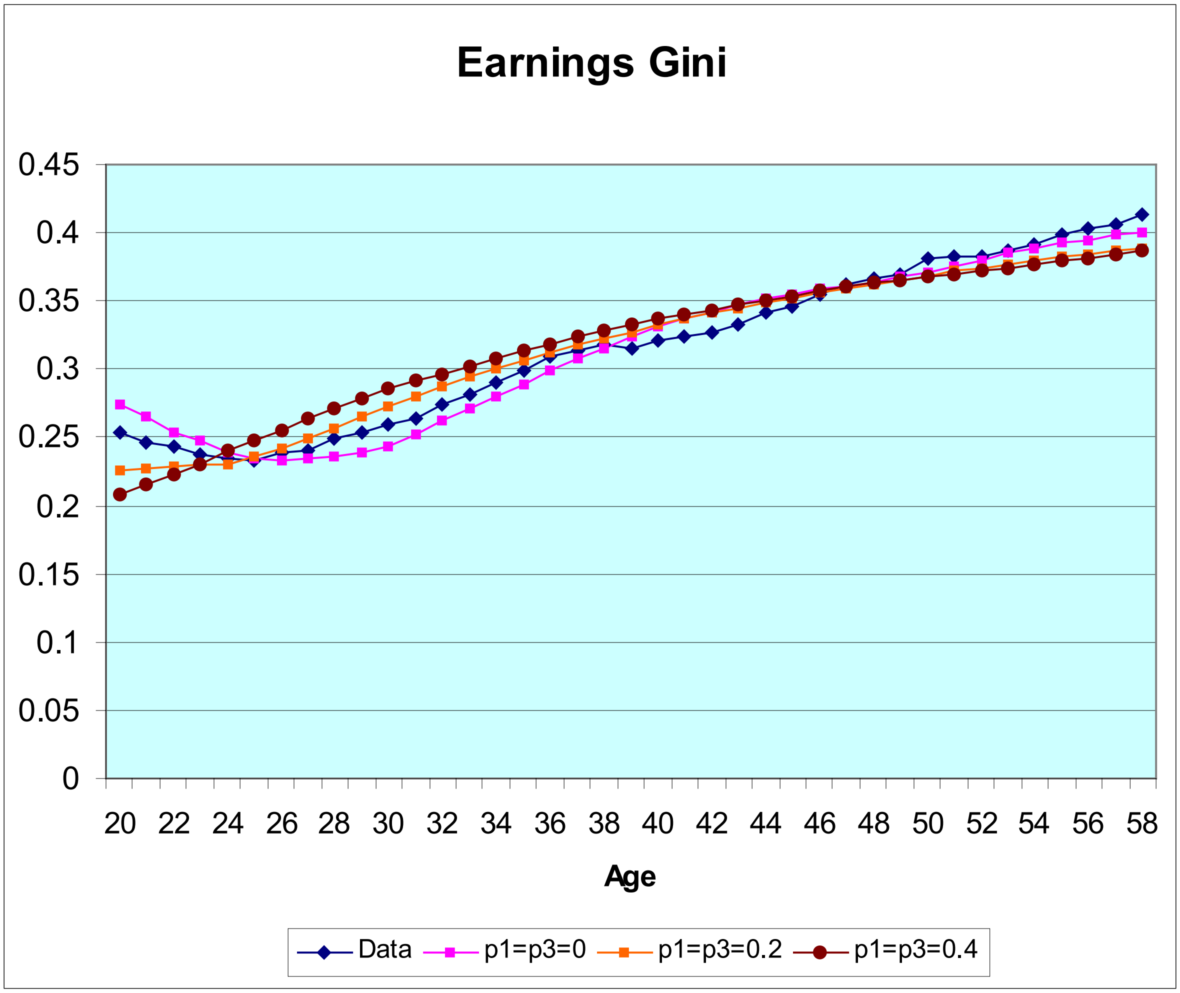
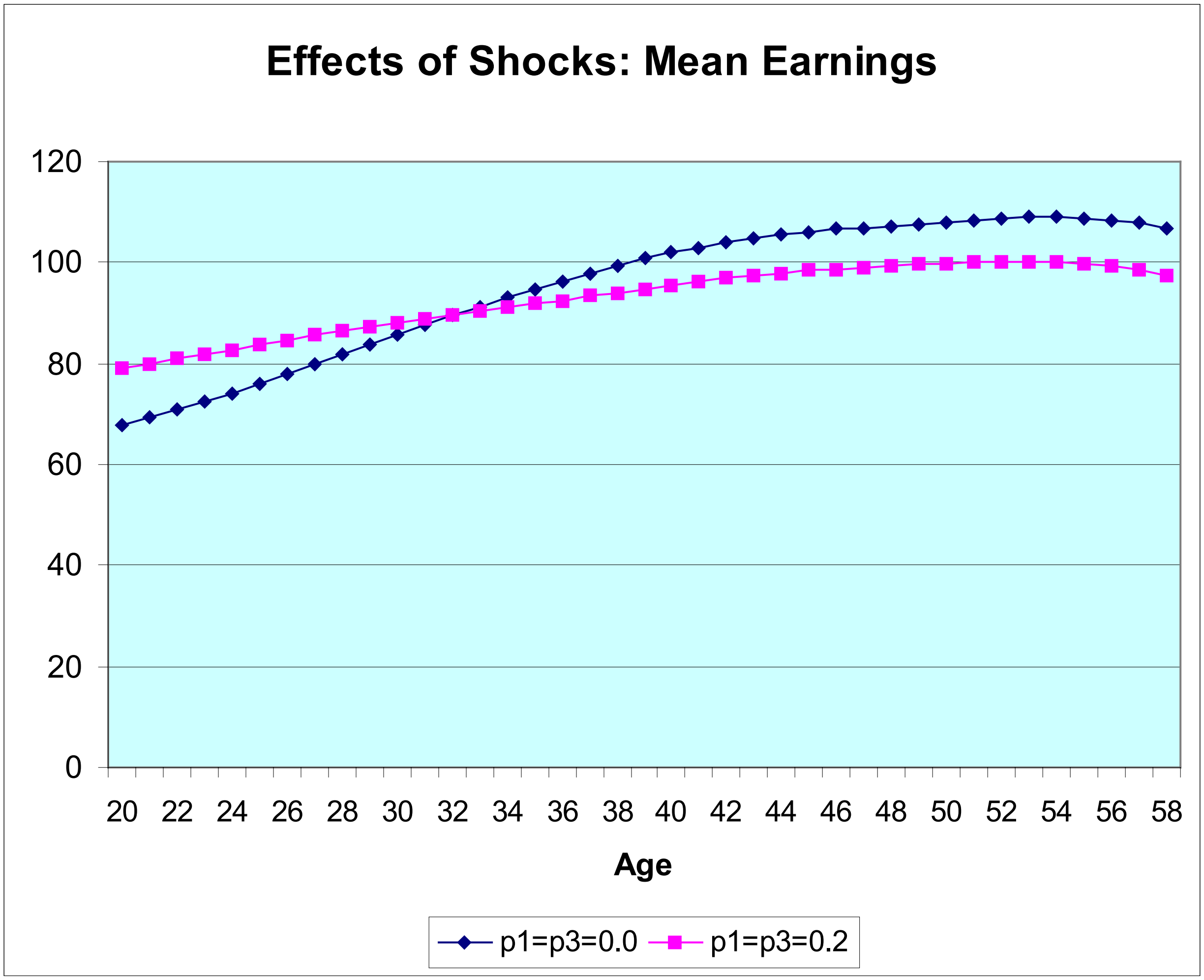

Earnings Gini
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58
Age
Data
p1=p3=0
p1=p3=0.2
p1=p3=0.4

Figure 2-a



Effects of Shocks: Mean Earnings
120
100
80
60
40
20
0
20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58
Age
p1=p3=0.0
p1=p3=0.2

Figure 2-b

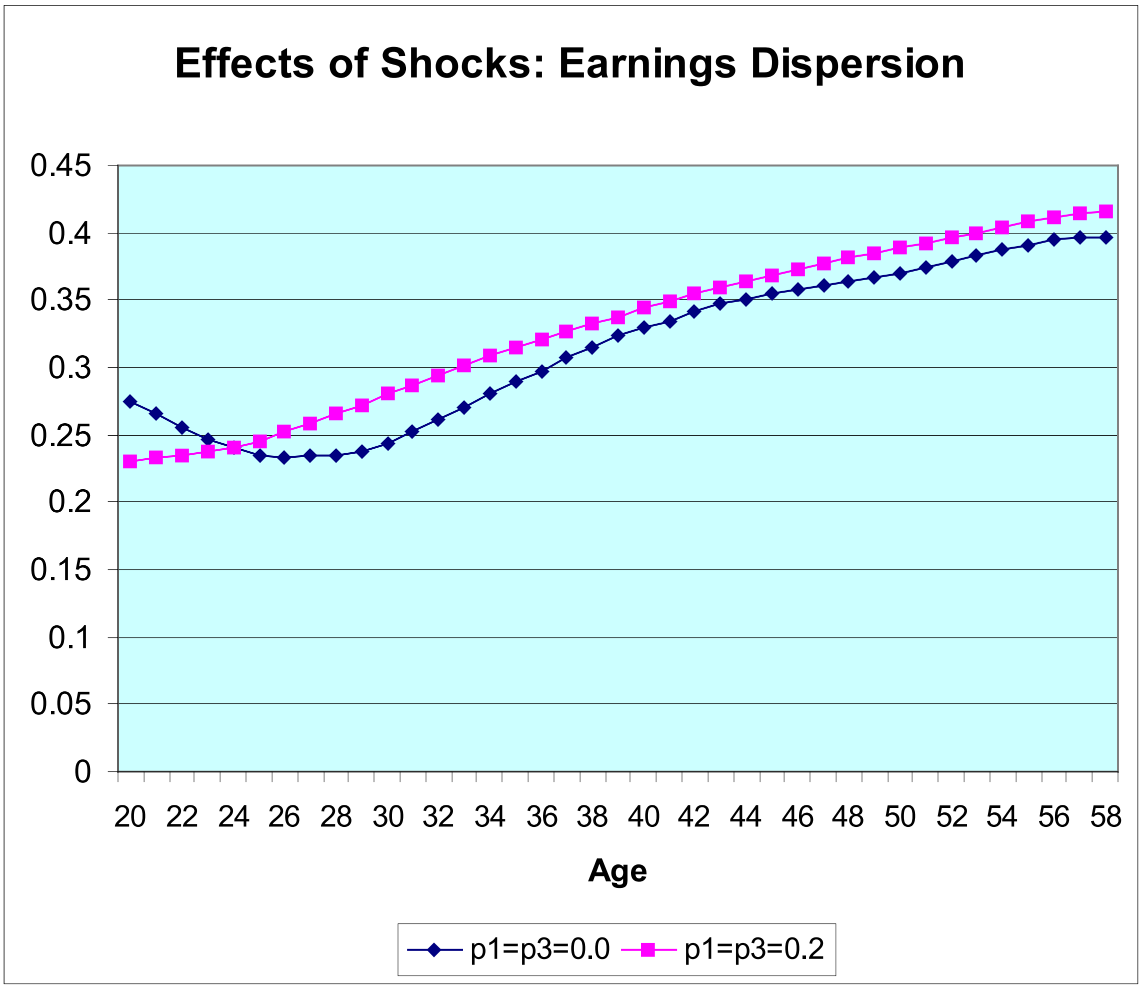
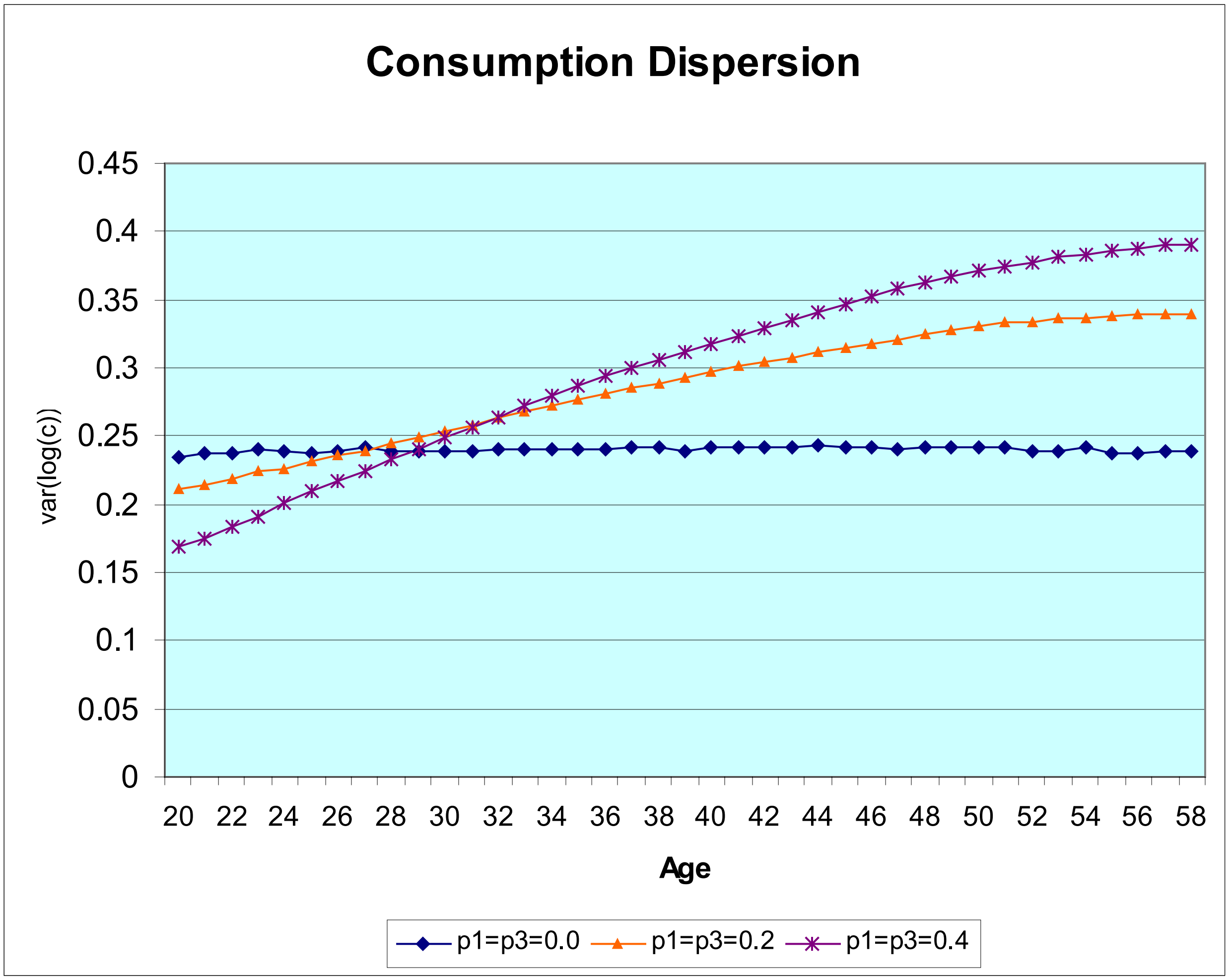

Effects of Shocks: Earnings Dispersion
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58
Age
p1=p3=0.0
p1=p3=0.2

Figure 3


Consumption Dispersion
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
var(log(c))
20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58
Age
p1=p3=0.0
p1=p3=0.2
p1=p3=0.4